VI *Curvarum Hyperbolicarum, æquationibus trium nominum utcunque definitarum, Quadratura generalis duplici Theoremate exhibita à Dº.* Samuele Klingenſtierna, *Profeſſ. Digniſſ. Math. in Acad.* Upſal, *& R. S. S. Communicante Dº.* Jacobo Stirling, *ejuſdem etiam Soc. Doctiſſ. S.*

N. B. *CURVÆ Hyperbolicæ, de quarum quadraturâ hic agitur ab Erud. Auctore, ad unum quaſi genus reducuntur, ex communi quâ gaudent proprietate, quantumvis obſcura ſit nec ſatis per ſe determinata. Ad hoc enim genus refertur, omnis curva, cujus ordinata datum efficit rectangulum cum recta, quæ ex tribus partibus neceſſario diverſis & ordine genitis conſtituitur. Diverſæ partes eſſe intelliguntur, quæ ex diverſis abſciſſæ poteſtatibus quomodocunque oriuntur; Ordine autem genitæ ſunt, ſi modo ab ima ad ſummam poteſtatem æquis gradibus aſcendant.*

Species igitur determinantur ac definiuntur ex gradibus Poteſtatum determinatis & definitis.

Primas & ſimpliciſſimas hujus generis (ad quas etiam ceteræ omnes ultimo reducuntur) Neutonus *ipſe primus ex datis Circuli & Hyperbolæ areis dimenſus eſt.*

Coteſius *deinde plures eſſe hujus generis Species, etiam in infinitum (ſecundum ordinem determinatum) progredientes detexit, quæ ad eandem quadraturæ*

turæ formam ac priores istæ & simpliciores reduci possint; idque fecit ope Theorematis cujusdam novi de Inventione radicum æquationum binomialium, ex determinata quadam divisione circumferentiæ Circuli in partes æquales; cujus Theorematis mentio facta est in Erudito suo Opere de Harmonia mensurarum.

Iisdem vestigiis insistendo D. Moivræus *Theorema Cotesianum ulterius promovit ad inventionem radicum æquationum Trinomialium, idque adhibendo arcum circuli determinatæ magnitudinis vice circumferentiæ totius. Quo invento omnes hujus generis Species inter se commensurabiles esse secundum rationem quadraturæ suæ statim perspexit, Methodumque tradidit in exquisitis suis scriptis* Miscellaneis *nuper editis, quâ perveniatur ad quadraturam unius cujus libet formæ ex datis Circuli & Hyperbolæ quadraturis.*

Ds. Kl. *in Propositione sua, quæ sequitur, in unum collegit quicquid de quadraturis curvarum hujus generis antehac a prioribus inventum fuit. Verum tamen ita collegit non quasi sint variæ formæ sub uno genere, sed quasi una sit eademque forma generis ipsius. Theorema duplex est, quatenus quadratura referat ad aream vel citra, vel ultra ordinatam. Exhibetur in ipsis æquationis terminis sine reductione aut restrictione. Instituitur secundum* Cotesii *doctrinam, usurpando mensuras Angulorum & Rationum pro areis Circuli & Hyperbolæ. Traditur sine demonstratione, utpote cujus veritas facile innotescat ex Propositionibus* Moivræanis.

Hæc

Hæc de hujus doctrinæ fontibus indigitasse, non abs re fore judicatum est, ne, lectoribus inexercitatis, auctoris nimia brevitas impedimento esset.

Propositio.

Quadrare curvam, cujus abſciſſa eſt z & ordinata $\frac{c z^{\eta \pm \frac{r}{n}\eta - 1}}{a^{2\eta} \pm a^{\eta-1} b z^{\eta} + z^{2\eta}}$, ubi η deſignat numerum quemlibet, r & n numeros quoſlibet integros & primos inter ſe, & denominator $a^{2\eta} \pm a^{\eta-1} b z^{\eta} + z^{2\eta}$ non poteſt reſolvi in duos factores binomios.

In circumferentia circuli (*Tab.* 2. *Fig.* 1.) centro quovis O intervallo $OR = a$ deſcripta applicetur chorda $RT = b$, cui parallelus ducatur radius O P, ita quidem ut arcus P R ſit quadrante major ſi habeatur $+ b$, minor vero ſi habeatur $- b$. Incipiendo in puncto R, ſumantur ordine tot arcus $R\overset{\mathrm{I}}{R}$, $\overset{\mathrm{I}}{R}\overset{\mathrm{II}}{R}$, $\overset{\mathrm{II}}{R}\overset{\mathrm{III}}{R}$, $\overset{\mathrm{III}}{R}\overset{\mathrm{IV}}{R}$, $\overset{\mathrm{IV}}{R}\overset{\mathrm{V}}{R}$, &c. arcui P R æquales, quot unitates continet fractio $\frac{r}{n}$, & a punctis $\overset{\mathrm{I}}{R}$, $\overset{\mathrm{II}}{R}$, $\overset{\mathrm{III}}{R}$, $\overset{\mathrm{IV}}{R}$, $\overset{\mathrm{V}}{R}$, &c. ducantur totidem rectæ $\overset{\mathrm{I}}{R}\overset{\mathrm{I}}{r}$, $\overset{\mathrm{II}}{R}\overset{\mathrm{II}}{r}$, $\overset{\mathrm{III}}{R}\overset{\mathrm{III}}{r}$, $\overset{\mathrm{IV}}{R}\overset{\mathrm{IV}}{r}$, $\overset{\mathrm{V}}{R}\overset{\mathrm{V}}{r}$, &c. radio O P parallelæ & rectæ O R occurrentes in punctis, $\overset{\mathrm{I}}{r}$, $\overset{\mathrm{II}}{r}$, $\overset{\mathrm{III}}{r}$, $\overset{\mathrm{IV}}{r}$, $\overset{\mathrm{V}}{r}$, &c. Deinde dividatur arcus P R in tot partes æquales quot ſunt unitates in numero η, quarum illa quæ puncto P ad-

adjacet sit P A. Facto initio in puncto A dividatur integra circumferentia in tot partes æquales A B, B C, C D, D E, &c. quot sunt unitates in n; sumtaque in radio O P, producto si opus ultra P, abscissa $OS = a.\overline{\frac{z}{a}}|^{\frac{n}{n}}$, jungantur S A, S B, S C, S D, S E, &c. ut & O A, O B, O C, O D, O E, &c. Denique sumantur arcus P A a, P B b, P C c, P D d, P E e. &c. qui sint ad arcus P A, P B, P C, P D, P E, &c. ut $n + r$ ad unitatem, & a punctis a, b, c, d, e, &c. ducantur tum rectæ $a\alpha$, $b\beta$, $c\gamma$, $d\delta$, $e\varepsilon$, &c. parallelæ radio O P & occurrentes rectæ O R in punctis α, β, γ, δ, ε, &c. tum etiam rectæ $a\,1$, $b\,2$, $c\,3$, $d\,4$, $e\,5$, &c. prioribus normales, & rectæ Q O, quæ ad R O ducatur perpendicularis, occurrentes in punctis 1, 2, 3, 4, 5, &c.

His factis area curvæ cujus abscissa est z & ordinata $\dfrac{cz^{n+\frac{r}{n}n-1}}{a^{2n} \pm a^{n-1}bz^{n} + z^{2n}}$, erit $\dfrac{nc}{na^{n+1}}z^{\frac{r}{n}n}$ in

$$\frac{\overset{\mathrm{I}}{R}\,\overset{\mathrm{I}}{r}}{r-n} \times \overline{\frac{a}{z}}\Big|^{n} - \frac{\overset{\mathrm{II}}{R}\,\overset{\mathrm{II}}{r}}{r-2n} \times \overline{\frac{a}{z}}\Big|^{2n} + \frac{\overset{\mathrm{III}}{R}\,\overset{\mathrm{III}}{r}}{r-3n} \times \overline{\frac{a}{z}}\Big|^{3n}$$

$$- \frac{\overset{\mathrm{IV}}{R}\,\overset{\mathrm{IV}}{r}}{r-4n} \times \overline{\frac{a}{z}}\Big|^{4n} - \frac{\overset{\mathrm{V}}{R}\,\overset{\mathrm{V}}{r}}{r-5n} \times \overline{\frac{a}{z}}\Big|^{5n}, \ \&c.$$

$$+ \frac{c}{n} a^{\frac{r}{n}n - n - 1} \text{ in} \left\{ \begin{array}{l} - a\alpha\,(SA:AO) - a1\,(+SAO) \\ + b\beta\,(SB:BO) + b2\,(+SBO) \\ - c\gamma\,(SC:CO) + c3\,(+SCO) \\ + d\delta\,(SD:DO) - d4\,(-SDO) \\ + e\varepsilon\,(SE:EO) + e5\,(-SEO) \\ \quad\&c. \qquad\qquad\qquad \&c. \end{array} \right\}$$

Et

Et area curvæ cujus abſciſſa eſt z & ordinata

$$\frac{c z^{n - \frac{r}{n} n - 1}}{a^{2n} \pm a^{n-1} b z^{n} + z^{2n}}, \text{ erit } \frac{nc}{na^{n+1}} z^{-\frac{r}{n} n} \quad \text{in}$$

$$\frac{\overset{\mathrm{I}}{R}\,\overset{\mathrm{I}}{r}}{r-n} \times \left.\frac{z}{a}\right|^{n} - \frac{\overset{\mathrm{II}}{R}\,\overset{\mathrm{II}}{r}}{r-2n} \times \left.\frac{z}{a}\right|^{2n} + \frac{\overset{\mathrm{III}}{R}\,\overset{\mathrm{III}}{r}}{r-3n} \times \left.\frac{z}{a}\right|^{3n}$$

$$- \frac{\overset{\mathrm{IV}}{R}\,\overset{\mathrm{IV}}{r}}{r-4n} \times \left.\frac{z}{a}\right|^{4n} - \frac{\overset{\mathrm{V}}{R}\,\overset{\mathrm{V}}{r}}{r-5n} \times \left.\frac{z}{a}\right|^{5n}, \text{ \&c.}$$

$$+ \frac{c}{n} a^{-\frac{r}{n} n - n - 1} \text{ in } \left\{ \begin{array}{l} -a\alpha(SA:AO) - a1(+ASO) \\ +b\beta(SB:BO) + b2(+BSO) \\ -c\gamma(SC:CO) + c3(+CSO) \\ +d\delta(SD:DO) - d4(-DSO) \\ +e\varepsilon(SE:EO) + e5(-ESO) \\ \quad \text{\&c.} \qquad\qquad\qquad \text{\&c.} \end{array} \right\}$$

Harum arearum prior adjacet abſciſſæ ad ordinatam terminatæ, poſterior vero abſciſſæ ultra ordinatam productæ. Signa autem quantitatum has expreſſiones ingredientium ita determinantur: 1. Rectæ $\overset{\mathrm{I}}{R}\,\overset{\mathrm{I}}{r}$, $\overset{\mathrm{II}}{R}\,\overset{\mathrm{II}}{r}$, $\overset{\mathrm{III}}{R}\,\overset{\mathrm{III}}{r}$, &c. afficiuntur ſignis affirmativis, ſi a punctis circumferentiæ $\overset{\mathrm{I}}{R}$, $\overset{\mathrm{II}}{R}$, $\overset{\mathrm{III}}{R}$, tendunt ſecundum directionem O P, negativis vero ſi ab iiſdem punctis ſecundum directionem contrariam P O procedunt. 2. Moduli rationum $a\alpha$, $b\beta$, $c\gamma$, &c. ſigna habent affirmativa, ſi a punctis a, b, c, &c. tendunt ſecundum directionem O P, negativa ſi ſecundum contrariam. 3. E centro circuli O

 cadat

cadat in chordam R T normalis O H. Et moduli angulorum *a* 1, *b* 2, *c* 3, *&c.* signis gaudebunt affirmativis si a punctis *a*, *b*, *c*, *&c.* tendunt secundum directionem H O, negativis si secundum contrariam. 4. Producatur radius P O donec circumferentiæ denuo occurrat in *p*, & anguli S A O, S B O, S C O, *&c.* ut & A S O, B S O, C S O, *&c.* sumi debent affirmative si existunt in semicirculo superiore P R *p*, negative si in inferiore. Et secundum has regulas signa quantitatum quibus areæ exprimuntur nostræ figuræ accommodavimus.

VII. *Casus rarissimus* Plicæ Polonicæ *enormis à D.* Abrahamo Vatero, *M. D. Prof. Anatom.* Wittemberg. *& R. S. S. per D.* Conradum Sprengell, *Equitem, M. D. R. S. S. & Coll. Med.* Lond. *Licent. communicatus.* Vid. Tab. II. Fig. 2.

FŒMINA rustica in Polonia, in terris Principis Radzivil, anno ætatis decimo quinto, viro nupta, incidit decimo octavo, in morbum Poloniæ Endemium, qui Plica Polonica a capillo inenodabili vocatur. Hanc Plicam per quinquaginta annos fœmina gestavit, ac per totum fere illud tempus dolore arthritico et contracturis tandemque marasmo universali corporis afflicta tecto affixa fuit, tandemque senio confecta anno ætatis septuagesimo octavo diem suum obiit.

Fig. 2.

Fig. 1.